ERRATUM

Magnetic Resonance in Medicine 58:211–212 (2007)

Erratum to Kellman P, McVeigh ER. Image reconstruction in SNR units: a general method for SNR measurement. Magn Reson Med. 2005;54:1439–1447.

SUMMARY OF REVISION

In the above article, a noise prewhitening approach was used prior to image reconstruction. In this approach, the noise correlation matrix becomes the identity defined with unit variance for real and imaginary noise components. Hence, the noise correlation $\mathbf{R}_\mathrm{n}$ did not appear explicitly in Eq. [7] as required. In this erratum Eq. [7] is rewritten to include the noise correlation. This step required modifying the definition of noise correlation (Eq. [1]) to be consistent with parallel imaging literature [Eq. 5], which led to further modification of Eqs. [5, 6]. The results are not affected since the experimental validation used the alternative noise prewhitening approach. The updated equations include the prewhitened form as Eq. [8]. The revised Eq. [1] and rewritten subsection Array Combining follow.

NOISE MEASUREMENT

Equation [1] is revised to conform with the more familiar conjugate form.

$$R_{ij} = (1/N)\sum_{k=1}^{N} n_i(k)n_j^*(k) \qquad [1]$$

and includes noise power contribution from real and imaginary components.

ARRAY COMBINING (*NOTE:* THIS REPLACES THE ARRAY COMBINING SECTION IN ITS ENTIRETY)

Roemer et al. (7) formulated equations for phased array combined image reconstruction for both root-sum-of squares (RSS) magnitude and optimum B_1-weighted combining, and Pruessmann et al. (5) formulated equations for parallel imaging using the image domain sensitivity encoding (SENSE) method.

$$SNR_{RSS} = \sqrt{2(\mathbf{p}^H\mathbf{R}_n^{-1}\mathbf{p})} \qquad [5]$$

$$SNR_{B1\text{-}weighted} = \sqrt{2}|\mathbf{b}^H\mathbf{R}_n^{-1}\mathbf{p}|/\sqrt{\mathbf{b}^H\mathbf{R}_n^{-1}\mathbf{b}} \qquad [6]$$

$$SNR_{SENSE} = \sqrt{2}|\mathbf{u}^T\mathbf{p}|/\sqrt{\mathbf{u}^T\mathbf{R}_n\mathbf{u}^*} \qquad [7]$$

The equations for signal-to-noise ratio (SNR) scaled images (Eqs. [5] and [6]) follow Roemer et al.'s (7) formulation, where *SNR* is the pixel intensity in SNR units, **p** is the vector of complex image values for each coil, **b** is the vector of complex coil sensitivities, and $\mathbf{R}_\mathrm{n}$ is the noise correlation matrix. The SNR estimates for the resultant magnitude images are very good at high SNR and may be further corrected to provide a good estimate at low SNR (1,2). The factor of $\sqrt{2}$ is used to account for the SNR definition in terms of the real channel noise component, consistent with MRI literature (1,2).

In the case of SENSE (5), described by Eq. [7], **u** represents the $\mathrm{N_{coils}}$x1 column vector of unmixing coefficients which are a reformatting of the unmixing matrix $\mathbf{U} = (\mathbf{S}^H\mathbf{R}_n^{-1}\mathbf{S})^{-1}\mathbf{S}^H\mathbf{R}_n^{-1}$ which contains the optimum noise weighting (**S** is the coil sensitivity matrix, superscript ^{H} denotes the Hermitian or conjugate transpose operation) (i.e., column vector **u** is transpose of *k*-th row of **U** for the *k*-th subimage). The unmixing vector is reformatted to be applied as a phased array combiner to the full field-of-view (FOV) images reconstructed with zero-filling of undersampled data. In the case of SENSE (5), Eq. [7] may be written equivalently as $\mathrm{SNR_{SENSE}} = \sqrt{2}|\mathbf{u}^T\mathbf{p}|\sqrt{(\mathbf{S}^H\mathbf{R}_n^{-1}\mathbf{S})_{k,k}}/g_k$ with the g-factor [5] for k-th subimage computed as $\mathrm{g(x,y + kFOV/R) = g_k(x,y)} = \sqrt{(\mathbf{S}^H\mathbf{R}_n^{-1}\mathbf{S})^{-1}_{(k,k)}(\mathbf{S}^H\mathbf{R}_n^{-1}\mathbf{S})_{(k,k)}}$, $0 < \mathrm{y} < \mathrm{FOV/R}$, and k = 0,1, . . . R–1, with R aliased images from uniform undersampling by R. In the preceding, the complex coil sensitivities $\mathrm{s_i(x,y)}$ are assumed to be known. Sensitivity estimates may be made using in vivo images acquired separately (5), or adaptively from the images in an autocalibrating manner (8–10).

Equations [5–7] describe the array combining that incorporates the noise weighting. Pixel intensities in SNR units are calculated by using the noise weighting as described along with scaling the signal to preserve the same effective gain as the noise. Equivalently, a noise prewhitening step may be applied by combining channels to create virtual channels, which are uncorrelated and have unit variance (11). Prewhitening may be realized by combining using the matrix $\mathbf{L}^{-1}$ (i.e., $\mathbf{p}_{\mathrm{prewhitened}} = \mathbf{L}^{-1}\mathbf{p}$), where the lower triangular matrix **L** is calculated from the Cholesky (11) factorization of the noise correlation matrix, $\mathbf{R}_\mathrm{n} = \mathbf{L}\,\mathbf{L}^H$. In the case of noise prewhitening, $\mathrm{SNR_{SENSE}}$ may be calculated as

$$SNR_{SENSE} = \sqrt{2}|\tilde{\mathbf{u}}^\mathrm{T}\mathbf{p}_{pre\text{-}whitened}|/\sqrt{\tilde{\mathbf{u}}^\mathrm{T}\tilde{\mathbf{u}}^*} \qquad [8]$$

where the noise correlation matrix for the whitened data becomes the identity, and $\tilde{\mathbf{u}}$ represents the vector of unmixing coefficients which are a reformatting of the unmixing matrix $\tilde{\mathbf{U}} = (\tilde{\mathbf{S}}^H\tilde{\mathbf{S}})^{-1}\tilde{\mathbf{S}}^H$, with the sensitivity matrix $\tilde{\mathbf{S}} = \mathbf{L}^{-1}\mathbf{S}$ corresponding to the virtual channels, i.e., following noise prewhitening. In the case of prewhitening, the denominator of Eq. [8] $\sqrt{\tilde{\mathbf{u}}^T\tilde{\mathbf{u}}^*}$ is equivalent to the g-factor,

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Received March 12, 2007; Accepted March 13, 2007.

DOI 10.1002/mrm.21261

i.e., $\mathrm{SNR}_{\mathrm{SENSE}} = \sqrt{2}|\tilde{\mathbf{u}}^T\mathbf{p}_{pre\text{-}whitened}|/g_k$, where it is assumed that relative sensitivities are used such that $diag(\tilde{\mathbf{S}}^H\tilde{\mathbf{S}}) = 1$, i.e., normalized by RSS combined magnitude ($\tilde{s}_i(x,y)/\sqrt{\Sigma\|\tilde{s}_i(x,y)\|^2}$, $\tilde{s}_i(x,y)$ is coil sensitivity profile for the ith virtual channel).

RESULTS

The results used the prewhitening form based on Eq [8]. The sentence regarding the image of Fig. 8a should be revised according to the new notation. "The image of Fig. 8a is scaled by all noise factors with the exception of the SENSE g-factor ($\sqrt{\tilde{\mathbf{u}}^T\tilde{\mathbf{u}}*}$)).“ Likewise for the SENSE implementation of Eq. [7], the output may be calculated as $\sqrt{2}|\mathbf{u}^T\mathbf{p}|\sqrt{(\mathbf{S}^H\mathbf{R}_n^{-1}\mathbf{S})_{k,k}}$ with separate scaling by the g-map, g_k, to avoid local noise enhancement in cases of high g-factor.

The conclusions and significance of this work are still valid. We apologize for any inconvenience caused by these errors.