Digital hologram reconstruction of radio telescope data

Peter Kellman, Steve Leonard, and Eamon Barrett

ESL, Incorporated, 495 Java Drive, Sunnyvale, California 94086.

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Fourier relations between antenna aperture distributions and their far fields are well known for coherent illumination. Less familiar are those for imaging incoherent sources. Such methods involve interferometry and are the basis for radio astronomy. Stated briefly, the radio star brightness distribution to be determined can be obtained from Fourier transforming measurements made of the complex visibility. The complex visibility is a measure of the complex correlation (or mutual coherence) of the received energy; and, therefore, it is not surprising that it should be a Fourier pair with the actual object intensity distribution. This is known as the well-celebrated Van Cittert-Zernike theorem. In the special case of radio astronomy, the 2-D correlation function is derived from a discretely spaced antenna array, where the vector spacing (measured by wavelengths) determines the spatial frequency. Many spatial frequencies are sampled as the earth rotates. The reconstruction of the source distribution from these samples is named rotational synthesis, which is the subject of this Letter.

The Fourier transform relation that has been quoted may be exploited for this purpose, but it is important to note that other methods to estimate the source distribution do exist and may have better properties since such methods (maximum entropy, for instance) account for noise and other errors neglected in a simple analysis. The outstanding advantage of the Fourier method is in its directness and ease of implementation. This Letter will deal with two such implementations of the Fourier transform: optical and digital.

The data used in our experiment consist of observations of the double radio source Cygnus-A gathered by the Stanford five-element array. The reconstruction steps are as follows: complex visibility measurements or correlations are recorded from each antenna pair and must be formatted to provide discrete samples of the 2-D complex spectrum of the radio source. Use of a discrete grid involves spatial quantization, which can be quite severe. The sampling grid in the spatial frequency domain is shown in Fig. 1. The digital reconstruction is simply derived from an FFT of the 2-D matrix of complex observations on this grid. The matrix size was 1024 $\times$ 1024. The resultant digital reconstruction is shown in Fig. 2. The reconstruction shows two point sources convolved with the synthetic aperture antenna pattern, which gives rise to the ring lobe structure.

The experimental procedure for the optical Fourier transform used in this work required a film recording of the input data. Furthermore, since the film transparency has only real and positive transmittance, it was necessary to code the complex data prior to recording. There are several computer holographic methods of coding. In these experiments we used a binary coded phase detour kinoform[3] (with phase quantization into eight bins). A real, positive matrix was recorded with a PDS microdensitometer in write mode. A positive transparency was then illuminated with coherent light from an argon laser, and the resultant spectral components were focused with a simple spherical lens onto film. The output plane recording is shown in Fig. 3.

A qualitative comparison between the optical and digital methods shows that the relatively simple optical analog technique produced a reconstruction in which two sources are

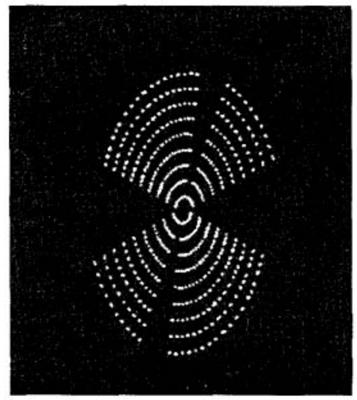

Fig. 1. Sampling grid in spatial frequency domain.

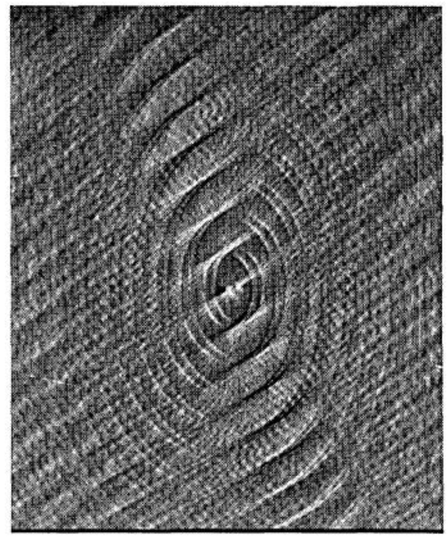

Fig. 2. Digital reconstruction of Cygnus-A.

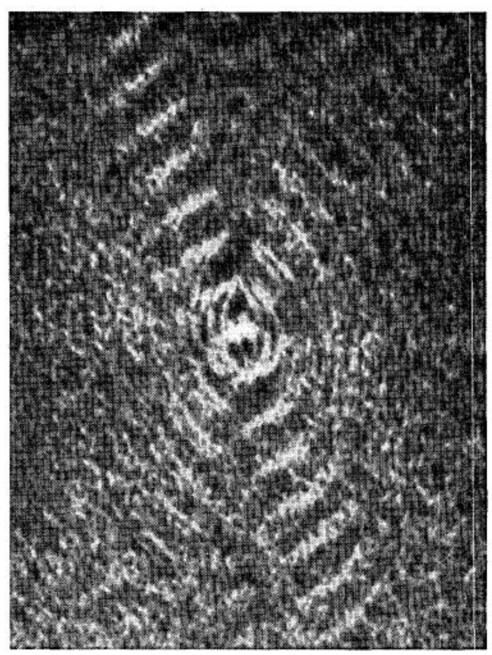

Fig. 3. Holographic reconstruction of Cygnus-A.

clearly resolvable. Other computer holographic methods would yield further improvement. Optical processing may be necessary to meet the enormous data processing requirements of future radio telescopes—for example, the National Radio Astronomy Observatory Very Large Array presently under construction in New Mexico.

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2. L. E. Somers and W. H. Carter, *1976 International Optical Computing Conference* p. 32.
3. Binary Lohmann hologram of phase information only.

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Diocles: On Burning Mirrors. G. J. TOOMER, Editor and English translator. Springer-Verlag, New York, 1976. 249 pp. $27.90.

A review by **Orestes N. Stavroudis**

One of the most fruitful periods in the history of science coincides with the classical era of ancient history. The era begins perhaps with Thales of Miletus, a legendary figure who is said to have predicted a total eclipse that occurred on 28 May 585 B.C., which, incidently, interrupted a battle between the Medes and the Lydians. A convenient date for the end of the era could be the murder of the woman mathematician Hypatia by a mob of followers of St. Cyril in 415 A.D. The *coup de grace* was administered by Justinian in 529 A.D. when he closed the academy at Athens because it was pagan.

Dirk J. Struik [*A Concise History of Mathematics* (Dover, New York, 1948)] divides this 1000-year long era into three periods, each with its own philosophies and each terminated by a series of crises. The first period is associated with Pythagoras and Hippocrates of Chios, the latter having written the only surviving fragment from this period. It shows that an extensive system of formal deduction had already developed. Its crises were Zeno's paradox, the discovery of incommensurables, and the Peloponesian wars. The Greek number system, incidently, is highly overrated. Although better than the Roman system of strokes and crosses, it was clearly inferior to the Babylonian sexagesimal system, which was not introduced into Greek mathematics until the second century A.D. by Claudius Ptolemy.

The second period begins with the development of a Greek aristocracy and aristocratic ideals as exemplified in the writings of Plato and Aristotle. As Struik put it: "mathematics had become a hobby of a leisure class, basing itself on slavery, indifferent to invention and interested in contemplation." The crises of the earlier period were resolved by Eudoxos's exhaustion method and a theory of irrational numbers. The most productive segment of this period (c.350–c.200 B.C.) found the greatest intellectual activity in Alexandria and not in Greece or Mesopotamia. To this period belongs Euclid of the Elements; Archimedes, who needs no introduction; and a much smaller light, Diocles.

The second period ended with the fall of Carthage, the subsequent collapse of Egypt and Greece, and the beginning of the third period, the era of Roman domination. Again quoting Struik, "originality and stimulation gradually disappeared, the *pax Romana*, lasting for many centuries allowed undisturbed speculation along traditional lines." Ptolemy introduced the sexagesimal number system. Diaphantos, whose equations inspired Fermat, and Pappus, of the theorem fame, both flourished. But the spark was gone and darkness fell. After a hiatus of several centuries the Greek–Egyptian scientific tradition was revived and continued by Arabic scholars who preserved and cherished its works. Many were translated into Arabic. In many cases the original manuscripts were lost. One such of these is Diocles "On Burning Mirrors."

Diocles on Burning Mirrors (the first volume in the new Springer series: Sources in the History of Mathematics and Physical Sciences) is the first of a promised series of source books in the history of mathematics and physics; the appearance of this book is a historical event of another kind. It marks the discovery of an Arabic translation of a long lost Greek tract on the geometry of conic sections. The Arabic manuscript was found in the Shrine library in the city of Meshhed, Iran. Toomer speculates that it arrived there as booty from the sack of Delhi by Nadir Shah in 1739. Strangely enough, a copy of this Arabic manuscript was subsequently found in the Chester Beaty Library in Dublin.

The book begins with some thirty pages of introductory material providing some historical background as well as a description of the found manuscript and an evaluation of the consequences of its discovery. I found this to be the most interesting portion. The Arabic text and Toomer's page by page translation, on facing pages, take up the next third of the volume. Next come photographs of the pages of the Shrine manuscript. This is followed by some thirty-five pages of notes and commentary.

The first of the four appendices consists of the Greek text and an English translation, again page by page and on facing pages, of a copy of a fragment of Diocles's text, heavily edited and amended by Eutocius in the sixth century A.D. Until the discovery of the Shrine manuscript this was the only known version. The second appendix contains two short proofs of the focal property of parabolas, one from the Classical period and one from the later Arabic era. The final two appendices consist of commentary and a pair of formal proofs by O. Neugebauer.