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SENSE Coefficient Calculation using Adaptive Regularization

Peter Kellman and Elliot R. McVeigh

Laboratory of Cardiac Energetics
National Heart, Lung and Blood Institute
National Institutes of Health
Bethesda, Maryland 20892-1061

kellman@nih.gov (301) 496-2513
emcveigh@nih.gov (301) 496-1184

Introduction

The SENSE method for accelerated MR imaging [1] is based on a least squares inverse solution that is frequently ill-conditioned, which results in a loss in SNR due to variance inflation. The SNR loss factor due to ill-conditioning, relative to phased array combining [2] for optimum SNR without acceleration, is referred to as the G-factor. Ill-conditioning of the inverse solution is caused by the lack of independence of the coil sensitivity profiles, a condition also referred to as collinearity. Ill-conditioning also increases the sensitivity to errors in the complex B1-maps. A technique known as regularization [3] or conditioning may be used to increase the tolerance to error and reduce the mean squared error. One such method for regularization is diagonal loading, also known in statistical literature as ridge regression [4]. This method is used in adaptive antenna array processing [5,6], and has been used for both SMASH [7] and SENSE [8,9] parallel MR methods. The method of diagonal loading may be used to tradeoff artifact suppression for a reduction in SNR loss (improved G-factor). In the TSENSE method [9], the increased artifact suppression achieved by temporal filtering permits a more flexible tradeoff of SENSE artifact rejection for SNR.

In this paper, a method for computing the SENSE coefficients using adaptive regularization is described. It is desirable to suppress the alias artifact to a specified level below either the desired signal or noise, in order that the SNR of the resultant image is not further degraded by artifact. Since the desired and aliased image intensities vary spatially, the desired or minimum required artifact suppression varies correspondingly. In most applications of diagonal loading, a fixed value of loading is used [7,8]. Using a fixed value of the regularization parameter results in artifact suppression that is spatially varying. However, in general there will be regions with artifact suppression that exceeds the minimum required suppression. In these regions, an increased value of diagonal loading may be used to reduce the SNR loss due to ill-conditioning. A method is described which adaptively adjusts the regularization parameter, based on an estimate of the desired artifact suppression, in order to meet a fixed signal-to-artifact ratio. Alternatively, given an estimate of the SNR, the adaptive regularization may be adjusted for a fixed artifact-to-noise ratio.

When the diagonal loading is zero, the least squares SENSE solution optimizes the SNR subject to the constraint of perfectly nulling the aliased component. As the diagonal loading becomes large, there is no suppression and the solution becomes the same as phased array combining for optimum SNR [8]. Thus, by using adaptive regularization, regions where the artifact is small relative to the desired image intensity achieve nearly optimum SNR (i.e., $G \approx 1$).

Methods

Description of Method

The regularized inverse solution for the SENSE coefficient matrix, **U**, may be calculated as:

$$\mathbf{U} = \mathbf{A}(\mathbf{S}^H \mathbf{R}_n^{-1} \mathbf{S} + \mathbf{\Lambda})^{-1} \mathbf{S}^H \mathbf{R}_n^{-1}, \tag{1}$$

where **S** is the matrix of coil sensitivities, $\mathbf{R}_n$ is the noise covariance matrix, $\mathbf{\Lambda}$ is a positive real diagonal matrix, which provides conditioning of the matrix inverse, **A** is a gain matrix (diagonal), and superscript ^{H} denotes the conjugate transpose (Hermitian) operator. In the case of regularization with fixed parameter, λ, i.e., $\mathbf{\Lambda}=\lambda\,\mathbf{I}$ with the identity matrix denoted by **I**, the value of λ is chosen to reduce the condition number of the matrix inverse by setting λ between the minimum and maximum eigenvalues ($\lambda_{min} < \lambda < \lambda_{max}$) of $\mathbf{S}^H\mathbf{R}_n^{-1}\mathbf{S}$, typically a small fraction of λ_{max}. As the parameter λ is increased, the solution approaches [8] the unconstrained phased array combiner for optimum SNR [2] (i.e., **U** proportional to $\mathbf{S}^H\mathbf{R}_n^{-1}$).

For the regularized SENSE matrix, the G-factor (SNR loss from optimum, neglecting $1/\sqrt{R}$ acceleration factor) may be calculated as:

$$G = \sqrt{\left(\mathbf{A}(\mathbf{S}^H\mathbf{R}_n^{-1}\mathbf{S}+\boldsymbol{\Lambda})^{-1}\mathbf{S}^H\mathbf{R}_n^{-1}\mathbf{S}(\mathbf{S}^H\mathbf{R}_n^{-1}\mathbf{S}+\boldsymbol{\Lambda})^{-1}\mathbf{A}\right)_{i,i}\left(\mathbf{S}^H\mathbf{R}_n^{-1}\mathbf{S}\right)_{i,i}}\,, \tag{2}$$

for the i-th sub-image, and the artifact suppression may be calculated from the off-diagonal elements of the matrix $\rho = \mathbf{U}\,\mathbf{S}$. For $\lambda=0$, ρ becomes the identity and the artifact is fully suppressed, assuming a perfect estimate of $\mathbf{S}$ (note that the gain matrix $\mathbf{A}=\mathbf{I}$ for $\lambda=0$). As λ is increased, the artifact suppression is degraded. The gain matrix, $\mathbf{A}$, is calculated such that the diagonal elements of ρ are equal to unity. In this case, for the rate R=2 example, the artifact suppression is ρ_{12} for the upper half of the image and ρ_{21} for the lower half.

The method of adaptive regularization described here uses a spatially varying regularization $\Lambda = \Lambda(x,y)$. For example, in the rate R=2 case,

$$\Lambda(x,y) = \begin{bmatrix} \lambda_{11}(x,y) & 0 \\ 0 & \lambda_{22}(x,y) \end{bmatrix}, \tag{3}$$

where $\lambda_{11}(x,y)$ and $\lambda_{22}(x,y)$ are maximized subject to the constraint of achieving the desired artifact suppression at location (x,y), $\rho_{21}(x,y)$ and $\rho_{12}(x,y)$, respectively. A block diagram for calculating the SENSE matrix coefficients using adaptive regularization is shown in Figure 1.

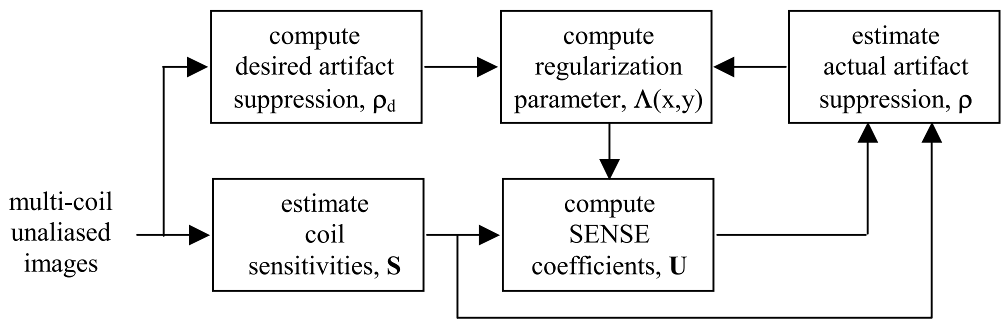

Figure 1. Block Diagram for Computing SENSE Coefficients with Adaptive Regularization.

SENSE coefficients may be calculated from the unaliased complex multiple coil images by a variety of methods. The TSENSE method [9] diagrammed in Figure 2 provides an adaptive means of obtaining lower temporal resolution unaliased images that may be used for estimating the raw sensitivities as well as the desired artifact rejection $\rho_d(x,y)$ used for adaptive regularization.

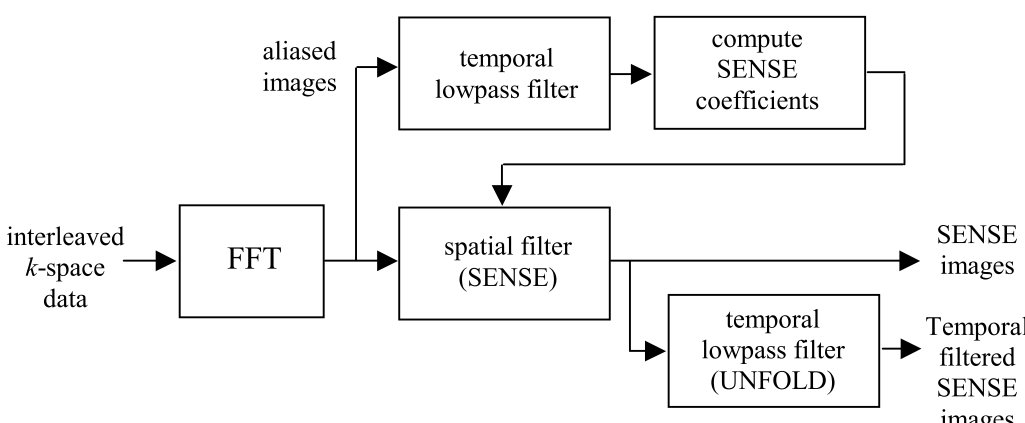

Figure 2. Block Diagram for Adaptive TSENSE.

Experimental Validation

Experiments were performed using a phantom to demonstrate the method and validate SNR performance. Imaging was performed using a GE Signa CV*i* 1.5T MR Imager. Data was reconstructed offline using the method described. A four-element cardiac phased array coil was used. The design of this array is not optimized for the SENSE processing application. Individual coils were rectangular with dimension of approximately 11.5 x 19 cm each, with the 4 coil array consisting of 2 overlapped coils (2 cm overlap along shorter side) on the top, and similarly 2 overlapped coils for the bottom pair. The phase encode direction was vertical. The field-of-view was 210mm x 210mm with an image matrix of 256 frequency encodes x 192 phase encodes. Axial slices of a cylindrical phantom were imaged repetitively, acquiring 100 images. SNR estimates were made from the sample mean and variance, which were calculated on a pixel-by-pixel basis in order to estimate SNR locally.

In this experiment, full *k*-space data was acquired, and R=2 SENSE data were generated by skipping every other phase encode line. G-factor estimates were made from the measured sample statistics, by ratio (pixel-by-pixel) of the SENSE image SNR with the SNR for B1-weighted full *k*-space reconstructions. SENSE coefficients were calculated using both fixed and adaptive regularization methods for comparison. The predicted G-factor and alias artifact suppression was also calculated from the raw sensitivity maps by direct evaluation of the analytic expressions.

Results

Figure 3 compares R=2 SENSE image reconstruction using fixed (top row) and adaptive regularization (bottom row). Figure 3 (a,d) are the images, Fig. (b,e) are estimates of the alias artifact suppression on a scale of 0 to 10%, and Fig. (c,f) are estimates of the G-factor on a scale of 1 to 1.25. The fixed regularization was chosen for a worst case artifact suppression of 1%. The adaptive regularization achieved the same 1% artifact suppression in the center where the artifact-to-signal ratio was greatest. The artifact suppression, Fig. 3(e), for the adaptive regularization method, is worst at the top and bottom where the artifact is smallest, and, conversely, is best where the artifact is greatest in the center.

The SNR of the SENSE images varied between 50 at edges and 10 near center. Using fixed regularization, the worst case G-factor was approximately 1.3 in the center and edges (measured with small ROIs). Using adaptive regularization, the G-factor was the same in the center, where a high level of artifact suppression was maintained, and was approximately 1.08 at the bottom edge. Note that the G-factor at the top of the image is low due to the presence of an air bubble in this region.

Regions without alias artifact do not require artifact suppression and, therefore, benefit from increased regularization. In these regions, nearly optimal SNR (G=1) is achieved without using thresholding, as seen in Fig 3(F), and corresponding to reduced suppression in Fig. 3(E) (bright areas with large value of ρ). Using fixed regularization without thresholding of noise only regions, results in increased noise as seen in Fig 3(C).

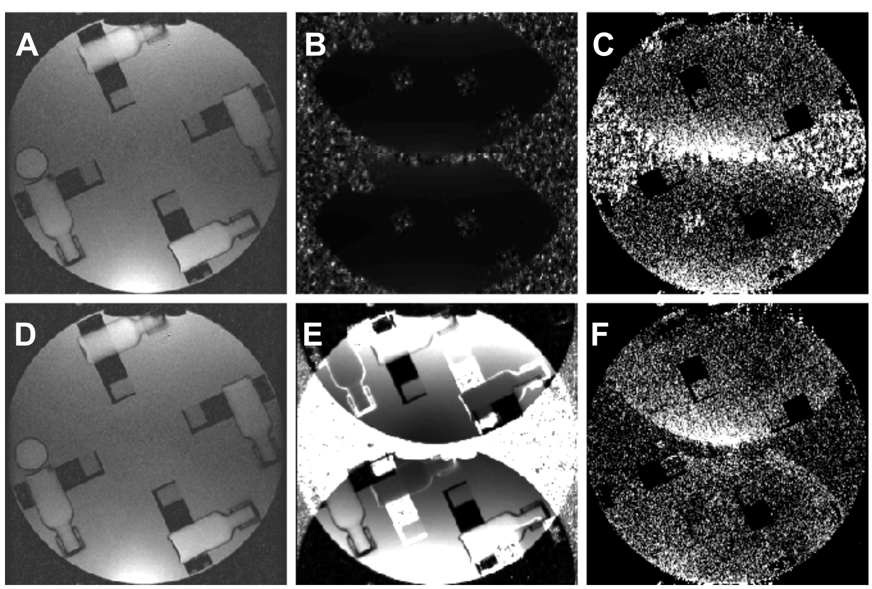

Figure 3. Comparison of R=2 SENSE reconstructions for fixed regularization (top row) and adaptive regularization (bottom row), showing images (A,D), artifact suppression (B,E) on scale 0-10%, and estimated G-factor (C,F) on scale 1-1.25.

Discussion

Preliminary results indicate potential benefits of this approach. Further evaluation is required to determine the sensitivity to noise and dynamic motion. The estimate of the gain matrix, **A**, may degrade the SNR if the raw images used to calculate sensitivities, **S**, are too noisy. Note, however, that the estimates of the sensitivity maps are significantly improved by both temporal and spatial smoothing.

For dynamic imaging with rapid motion there may be undesirable edge effects for which the estimated artifact suppression is too low. This can be mitigated to large extent by smoothing lambda using an order statistic filter, which uses the minimum lambda in the local neighborhood (designed to be greater than the maximum motion over the temporal smoothing window).

The required artifact suppression will depend on the application. In the TSENSE method, the increased artifact suppression achieved by temporal filtering permits a more flexible tradeoff of SENSE artifact rejection for SNR. Another benefit of the adaptive regularization is that it achieves the optimum SNR in regions without any artifact, without using a hard decision or threshold approach.

Conclusion

A method for adaptive regularization of SENSE coefficient calculation has been presented. Preliminary results are shown for a phantom with rate R=2 acceleration. This method has potential for improving the G-factor in specific areas such as the peripheral region that has relatively little artifact to cancel. This may be significant in applications such as fMRI of cortical regions [10]. It also has potential for ghost cancellation applications [11] applied to multi-shot EPI where adaptive regularization may be used to achieve a high degree of cancellation only in regions with large artifacts (e.g., off-resonance regions). Experimental evaluation is planned for several applications, as well as extension to higher acceleration rates.

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References

1. Pruessmann KP, Weiger M, Scheidegger MB, Boesiger P. SENSE: Sensitivity encoding for fast MRI. *Magn Reson Med* 1999; 42:952–962.
2. Roemer PB, Edelstein WA, Hayes CE, Souza SP, Mueller OM. The NMR phased array. *Magn Reson Med* 1990;16:192-225.
3. Tikhonov AN, Arsenin VY. *Solutions to ill-posed problems*. Winston, Washington 1977.
4. Hoerl AE, Kennard RW. Ridge Regression: biased estimation for nonorthogonal problems. *Technometrics* 42(1): 80-86. 2000.
5. Carlson BD, Covariance Matrix Estimation Errors and Diagonal Loading in Adaptive Arrays. *IEEE Transactions on Aerospace and Electronic Systems*, 1988; 24:397-401.
6. Cox H, Zeskind RM, Owen MM. Robust adaptive beamforming. *IEEE Trans. Acoustics, Speech, and Signal Proc*. ASSP-35(10): 1365-76. 1987.
7. Sodickson DK. Tailored SMASH image reconstructions for robust in vivo parallel MR imaging. *Magn Reson Med* 2000; 44:243-251.
8. King KF, Angelos L. SENSE image quality improvement using matrix regularization. *Proc. Ninth Scientific Meeting Intl. Soc. Magn Reson Med* 2001;1771.
9. Kellman P, Epstein FH, McVeigh ER, Adaptive sensitivity encoding incorporating temporal filtering (TSENSE). *Magn Reson Med*. 2001 May; 45(5):846-52.
10. de Zwart JA, Kellman P, van Gelderen P, Duyn JH. On the Potential of Sensitivity Encoded EPI for BOLD Functional Brain Imaging. *Proc. Ninth Scientific Meeting Intl. Soc. Magn Reson Med* 2001;1217.
11. Kellman P, McVeigh ER, Ghost Artifact Cancellation using Phased Array Processing. *Magn Reson Med*. 2001 Aug; 46(2):335-43.