Image Reconstruction in SNR Units:

A General Method for SNR Measurement

Peter Kellman and Elliot R. McVeigh

Laboratory of Cardiac Energetics, National Heart Lung and Blood Institute, NIH, DHHS, Bethesda, Maryland 20892 USA

INTRODUCTION

Reconstruction of uniform noise images directly in SNR units facilitates accurate SNR measurement on a per pixel basis. This method is applicable to root-sum-of-squares magnitude combining, B1-weighted combining, and parallel imaging. The purpose of this paper is to provide a nuts and bolts procedure for calculating scale factors used for reconstructing images directly in SNR units. Alternative methods that rely on noise only regions are not appropriate for parallel imaging where the noise level is highly variable across the field-of-view.

METHODS

Overview

A general diagram of image reconstruction is shown in Fig. 1. Each signal processing step may change the noise standard deviation. The overall scale may be computed for the end-to-end image reconstruction or, alternatively, each step may be scaled to maintain a unity noise gain for which the output has the same noise standard deviation as the input as diagrammed in Fig. 2. Several cases of array combining are treated:

(a) RSS combined magnitude,
(b) B1-weighted combining, and
(c) parallel imaging using the image domain SENSE method.

A summary of signal processing steps is shown in Fig. 3.

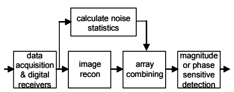


Figure 1. Image reconstruction steps.

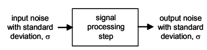


Figure 2. Individual signal processing step with unity gain for noise standard deviation.

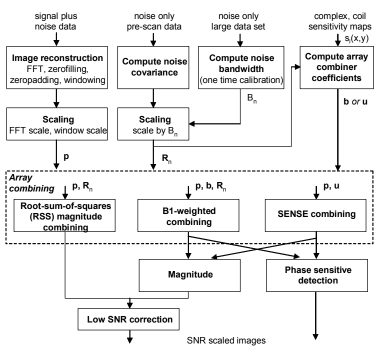


Figure 3. Procedure for SNR scaled image reconstruction (**p** is complex vector of multi-coil images, **b** is complex vector of coil sensitivities, **u** is complex vector of SENSE unmixing coefficients, $\mathbf{R}_n$ is noise correlation matrix, and B_n is noise equivalent bandwidth).

Noise Measurement

The noise covariance is measured in order to perform noise scaling and optimum array combining. The noise covariance matrix $\mathbf{R}_n$ may be estimated from N pre-scan noise-only samples acquired without RF pulses:

$$R_{ij} = (1/N)\sum_{k=1}^{N} n_i^*(k)\, n_j(k) \qquad (R_{ij} \text{ are components of } \mathbf{R}_n).$$

The noise covariance must be scaled by the noise equivalent bandwidth factor, B_n, since the input noise spectrum is not flat across the full bandwidth:

$$\mathbf{R}_{n\ scaled} = \mathbf{R}_n / B_n.$$

The noise equivalent bandwidth is easily measured by calculating the mean squared value of the average noise power spectrum (see Fig. 4) normalized by the response at the center.

$$B_n = (1/N)\sum_{k=1}^{N} |H(k)|^2 \Big/ |H(0)|^2$$

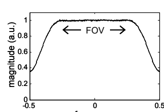


Figure 4. Average noise spectrum. 12288 averages with 2x readout oversampling. The noise equivalent receiver bandwidth factor is 0.79 for this example

FFT Based Image Reconstruction

FFT based image reconstruction may include windowing for reduced Gibb's ringing, zerofilling of missing data, and zeropadding for interpolation. A standard FFT

$$\sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

with input white noise having standard deviation σ will have standard deviation σ√N after FFT, thus a scale factor of 1/√N must be applied for unity noise gain (N is the number of actual data samples and not including zero-padding or zero-filling).

A window function, w(n), must likewise be scaled by its root-mean-squared value such that the input and output have the same noise standard deviation, σ , i.e.:

$$w_{scaled}(n) = w(n) \Big/ \sqrt{(1/N)\sum w^2(n)}$$

Array Combining

Roemer, et al [1] formulated equations for phased array combined image reconstruction for both root-sum-of squares (RSS) magnitude and optimum B1-weighted combining, and Pruessmann, et al. [2] formulated equations for parallel imaging using the image domain SENSE method.

$$SNR_{RSS} = \sqrt{\mathbf{p}^T \mathbf{R}_n^{-1} \mathbf{p}^*} \qquad [1]$$

$$SNR_{B1-weighted} = \left|\mathbf{p}^T \mathbf{R}_n^{-1} \mathbf{b}^*\right| \Big/ \sqrt{\mathbf{b}^T \mathbf{R}_n^{-1} \mathbf{b}^*} \qquad [2]$$

$$SNR_{SENSE} = \left|\mathbf{u}^T \mathbf{p}\right| \Big/ \sqrt{\mathbf{u}^T \mathbf{u}^*} \qquad [3]$$

Equations [1] and [2] for SNR scaled images follow Roemer's formulation, where *SNR* is the pixel intensity in SNR units, **p** is the vector of complex image values for each coil and **b** is the vector of complex coil sensitivities. In the case of SENSE [2], described by Eq. [3], **u** represents the vector of unmixing coefficients (a reformatting of the unmixing matrix **U)** which contains the optimum noise weighting (**S** is the coil sensitivity matrix).

$$\mathbf{U} = \left(\mathbf{S}^H \mathbf{R}_n^{-1} \mathbf{S}\right)^{-1} \mathbf{S}^H \mathbf{R}_n^{-1}$$

Magnitude Image Correction

In the case of magnitude detection, an SNR dependent noise correction [3,4] must be applied even though the true input noise standard deviation is used in scaling due to the inherent noise bias in magnitude detection. For example, at SNR=0 (noise only) the bias is 1.25σ, 1.88σ, 2.74σ, 3.94σ, 5.61σ, and 7.97σ for N_c= 1, 2, 4, 8, 16 and 32 channels, respectively. SNR dependent correction may be implemented by look-up-table [3,4]. Correction curves for low SNR magnitude correction are shown in Fig. 5 The correction factor is simply subtracted from the measured SNR.

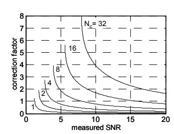


Figure 5. Low SNR magnitude correction curves (Nc = number of channels combined).

Experimental Validation

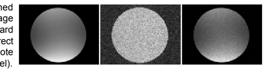
Validation of SNR scaled image reconstruction was performed using a time series of 256 phantom images acquired using a Siemens Sonata 1.5T scanner using a single-shot TurboFLASH sequence. Pre-scan noise and raw data were acquired and reconstruction was performed off-line using Matlab. The sample mean and standard deviation were computed for each pixel using the 256 image frames, and a direct estimate of the SNR was calculated as the ratio of sample mean to sample standard deviation. This direct SNR measurement was then compared to the mean of the SNR scale image reconstructed images which should be equivalent.

In-vivo cardiac images were acquired using the 32-channel Siemens 1.5T Avanto and a prototype 32-element cardiac array (Invivo Corp). Imaging of a normal volunteer was performed using a breath-held, segmented, ECG triggered, true-FISP cine sequence. Full k-space images were acquired and reconstructed using RSS magnitude combining, as well as SENSE accelerated images acquired at rates 2, 3, and 4, using the auto-calibrating TSENSE method. A single, doubly oblique, short-axis slice was acquired with phase encoding performed along the "AP" and "LR" directions for comparison (separate acquisitions).

RESULTS

The case of RSS magnitude combined imaging is shown in Fig. 6. The mean SNR scaled images of Fig. 6(a) agree with the SNR measurements of Fig. 6(c) within 4% in the phantom. Note that for magnitude images the standard deviation for noise only is less than 1, which results in the image appearing darker in the noise only region outside the phantom.

Figure 6. Root-sum-of-squares (RSS) combined magnitude image reconstruction example: (a) average SNR scaled image (256 trials averaged), (b) standard deviation of SNR scaled images (256 trials), (c) direct SNR estimate computed as ratio of (a) and (b), (note that (a) and (c) are displayed with same window-level).



The case of SENSE imaging (rate 2 using 4 coils) is shown in Fig. 7. SNR scaled reconstruction produces a uniform noise image, as evidenced by uniform standard deviation within the phantom (Fig. 7(b)). The uniform noise image Fig. 7(a) after 256 averages clearly shows the spatially dependent loss due to g-factor. The mean SNR scaled images of Fig. 7(a) agree with the SNR measurements of Fig. 7(c) within 5% in the phantom as measured at several locations. Fig. 8(a) shows a more standard reconstruction weighted by RSS magnitude of sensitivities without scaling by SENSE g-factor for uniform noise. In cases such as this with poor g-factor, the uniform noise image of Fig. 7(a) appears to have an artifact due to the local noise amplification, therefore, it may be preferable to output image of Fig 8(a) as well as g-map of Fig 8(c). Then the SNR scaled image of Fig. 7(a) may be derived as the ratio (Fig. 8(a) image divided by Fig. 8(c) g-map).

Figure 7. SENSE combined image reconstruction example: (a) average SNR scaled image (256 trials averaged), (b) standard deviation of SNR scaled images (256 trials), (c) direct SNR estimate computed as ratio of (a) and (b), (note that (a) and (c) are displayed with same window-level).

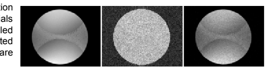
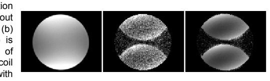


Figure 8. SENSE combined image reconstruction example: (a) average SNR scaled image without including g-factor term (256 trials averaged), (b) standard deviation of images (256 trials) in (a) is direct estimate of SENSE g-factor, (c) estimate of SENSE g-factor computed from complex coil sensitivities, (note that (b) and (c) are displayed with same window-level, 1 < g <1.4).



Example SNR scaled (uniform noise) cardiac short-axis images are shown in Fig. 9 Corresponding g-factor maps based on the coil sensitivity estimates are shown in Figure 10 for rates 2, 3, and 4, respectively. SNR measurements may be made by simply reading the pixel intensity. For example, at rate R=1 (i.e., without SENSE acceleration) the SNR in the myocardium of the left ventricle was measured as 30.9 for mid-septal and 22.0 for anterolateral with "AP" phase encoding (Fig. 9(a)), and 39.3 and 26.9 for corresponding locations with "LR" phase encoding (Fig. 9(e)). This is within 10% of expected after considering the √2 factor for acquisition time and ratio of voxel volumes.

Figure 9. Short-axis cardiac imaging example of SNR scaled uniform noise reconstructions using full k-space RSS combined magnitude and SENSE acceleration rates 2, 3, and 4 (left to right) with phase encoding direction AP (top row) and LR (bottom row). Note that the bottom row images have approximately √2x SNR due to the double acquisition time required for LR encoding to achieve approximately the same temporal and spatial resolution.

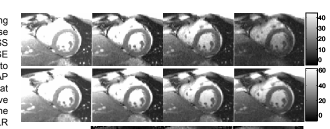
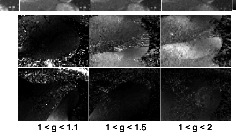


Figure 10. SENSE g-factor computed from raw complex coil sensitivities for rates 2, 3, and 4 (left to right) with phase encoding direction AP (top row) and LR (bottom row).



CONCLUSIONS

The proposed image scaling and SNR measurement method is broadly applicable to a number of image reconstruction methods and has a number of advantages over using a noise only region to estimate background noise. The use of pre-scan noise avoids contamination of the noise region by signal artifact, may use a large number of pre-scan noise samples which reduces the fluctuation of the noise estimate. The method may be used with parallel imaging such as SENSE for which the noise is locally varying due the g-factor. SNR measurement using SNR scaled images is both easy and accurate.

REFERENCES

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